Resource Allocation via Linear Programming for Fractional Cooperation

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***Abstract*— In this letter, resource allocation is considered for large multi-source, multi-relay networks employing fractional co-operation, in which each potential relay only allocates a fraction of its resources to relaying. Using a Gaussian approximation, it is shown that the optimization can be posed as a linear program, where the relays use a demodulate-and-forward (DemF) strategy, and where the transmissions are protected by low-density parity-check (LDPC) codes. This is useful since existing optimization schemes for this problem are nonconvex.**

I. Introduction

In wireless networks, spatial distribution of nodes generally results in independent fading on different links. This spatial distribution can be exploited in cooperative diversity [1], [2], where each node can assist its neighbours in transmitting information to a data sink. In its simplest form, a cooperative system is a relay system consisting of three nodes: a source, a relay, and a destination. The relay can use various cooperative schemes such as decode-and-forward (DF) [3] and demodulate-and-forward (DemF) [4], [5], to assist the source in transmitting its information bits to the destination.

In most wireless networks, a source node is typically in radio range of multiple relays. Fractional cooperation [6] is a low-complexity cooperative scheme for such multi-relay systems, often used in conjunction with DemF (though it can also be used with DF). Using this scheme, a large number of relays forward a small fraction of the source's transmission bits, so that the relaying cost is spread over a large number of relays. This scheme has good diversity order properties in fading channels.

A key challenge in fractional cooperation is *resource allocation*, in which the system determines what fraction must be selected for transmission by each relay. This challenge is exacerbated in systems with multiple sources, all of which are competing for the same fractional resource at the relays. In light of this challenge, this letter makes two conclusions:

- Resource allocation in multiple-source, multiple-relay fractional cooperation networks can be posed as an instance of linear programming. Our optimization minimizes the number of transmission bits (i.e., energy), subject to the constraint that decoding at the destination is successful. Our approach is particularly useful since efficient optimization algorithms are not known to exist for this problem; an earlier approach, based on the union bound, resulted in nonconvex optimization [7].
- The optimal resource allocations found by our method are *not*, in general, equivalent to selection: we give examples in which multiple sources and relays are assigned some fraction that is equal to neither its maximum nor minimum possible contribution. This suggests that there is not always a "best" relay for a source using fractional cooperation with DemF.

Our optimization strategy is similar to the use of linear programming to optimize low-density parity-check (LDPC) codes [8]: a Gaussian approximation is used, which make the objective and constraints all linear. Related work also includes [9]–[11], where extrinsic information transfer (EXIT) chart techniques [12] were used for code optimization for relays, not resource allocation.

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II. System Model

A. Channel Model

Our system uses DemF along with *fractional cooperation* (the reader is directed to [6] for complete details). In fractional cooperation, instead of forwarding the entire source's transmission sequence the relay nodes select *a random portion* of the source's transmission bits for relaying. Because of this random selection, some of the source's transmission bits will not be received at the destination. The source employs powerful error correcting codes to encodes its transmission bits, thereby ensuring successful reconstruction of its symbols at the destination despite the missing bits. Furthermore, DemF is used for its low complexity.

Consider s sources, r relays, and a single destination, as in Figure 1. The r relays are shared amongst all s sources. Let S_i and R_j represent the ith source and jth relay, respectively. Each source has a length-n information sequence to transfer to the destination represented by $\mathbf{x}^{(S_i)} = [x_1^{(S_i)}, x_2^{(S_i)}, ..., x_n^{(S_i)}]$, where $x_k^{(S_i)} \in \{0, 1\}$. Each source encodes its information sequence using an LDPC code. Let $\rho_1, \rho_2, \cdots, \rho_s$ be the code rates at each source. Therefore, the codeword ready for transmission at the ith source is represented by $\mathbf{z}^{(S_i)} = [z_1^{(S_i)}, z_2^{(S_i)}, ..., z_{m_i}^{(S_i)}]$, where $m_i = n/\rho_i$ is the length of the codeword.

We assume all links are independent additive white Gaussian noise (AWGN) channels, represented with their respective channel signal-to-noise ratio (SNR). (The channels are AWGN since the channel state information is assumed to be known, but we will consider cases where the SNR is random and arises from Rayleigh-distributed channel amplitudes.) There are s

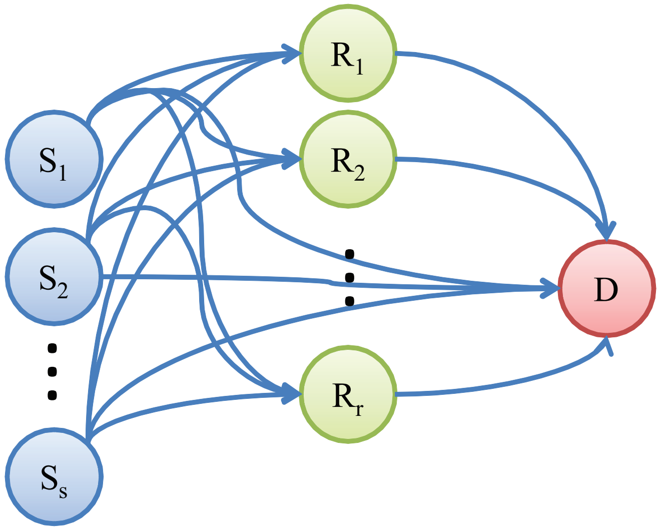

Fig. 1. Multi-Source, Multi-relay model.

source-to-destination (S-D) links, r relay-to-destination (R-D) links, and sr source-to-relay (S-R) links. We assume these communication links use binary phase-shift keying (BPSK) for data modulation. We define the function $\phi : \{0,1\} \rightarrow \{+1,-1\}$ as the modulation function where 0 is mapped to a +1 and 1 is mapped to -1. The S-D links are therefore given by

$$\mathbf{y}^{(S_i,D)} = \phi(\mathbf{z}^{(S_i)}) + \mathbf{n}^{(S_i,D)}, \tag{1}$$

where $\mathbf{n}^{(S_i,D)}$ is AWGN with variance $\sigma^2_{(S_i,D)}$. The channel SNRs for each of the s S-D links are represented by $\gamma_{(S_i,D)} = 1/(2\sigma^2_{(S_i,D)})$. The S-R links are also given by

$$\mathbf{y}^{(S_i,R_j)} = \phi(\mathbf{z}^{(S_i)}) + \mathbf{n}^{(S_i,R_j)}, \tag{2}$$

where S_i and R_j correspond to the ith source and the jth relay respectively and $\mathbf{n}^{(S_i,R_j)}$ is AWGN with variance $\sigma^2_{(S_i,R_j)}$. Therefore, all the S-R links can be represented by sr channel SNRs, $\gamma_{(S_i,R_j)} = 1/(2\sigma^2_{(S_i,R_j)})$.

In DemF, a relay first demodulates the signal received from a source. With slight abuse of the inverse notation, the demodulation function is defined as $\phi^{-1} : \mathbb{R} \rightarrow \{0,1\}$, where

$$\phi^{-1}(y) = \begin{cases} 0 & \text{if } y \geq 0 \\ 1 & \text{otherwise} \end{cases}, \tag{3}$$

Thus, the DemF process can be formulated as

$$\mathbf{z}^{(S_i,R_j)} = \phi^{-1}(\mathbf{y}^{(S_i,R_j)}), \tag{4}$$

where $\mathbf{z}^{(S_i,R_j)}$ is the results of hard decisions (demodulation) for the jth relay assisting ith source.

Each relay then selects a fraction of the demodulated signal, re-encodes it using error correcting codes and transmits to the destination. The vector $\mathbf{b}^{(S_i,R_j)}$ represents the demodulated bit positions selected for transmission to the destination: if $b_k^{(S_i,R_j)} = 1$, then the kth bit is relayed; if $b_k^{(S_i,R_j)} = 0$, then the kth bit is not relayed.

The vector $\mathbf{b}^{(S_i,R_j)}$, has a Hamming weight of $m_i\epsilon_{(i,j)}$, where $\epsilon_{(i,j)}$ is the fraction to be forwarded by the jth relay for the $i$the source. Thus, the relaying fraction $\epsilon_{(S_i,R_j)}$ is defined as

$$\epsilon_{(S_i,R_j)} = \frac{\sum_{i=1}^{m_i} \mathbf{b}^{(S_i,R_j)}}{m_i}. \tag{5}$$

This random selection of bits is similar to puncturing codes, and can be carried out using pseudorandom number generators.

Finally, transmission to the destination is encoded using a powerful and capacity-approaching error-correcting code. We will consider two cases in the sequel: first, we assume that decoding is successful at the destination, with an energy cost equal to the link capacity; and second, we use a particular punctured systematic repeat-accumulate (PSRA) code [6], with possible decoding failures. If the code is decoded successfully, the demodulated sequence resulting from the jth relay assisting ith source is available at the destination as

$$\mathbf{y}_{DemF}^{(S_i,R_j,D)} = \mathbf{b}^{(S_i,R_j)} \odot \phi(\mathbf{z}^{(S_i,R_j)}), \tag{6}$$

where $\odot$ is element-wise multiplication of vectors, $\mathbf{z}^{(S_i,R_j)}$ is given by equation (4), and $\mathbf{y}^{(S_i,R_j,D)}$ represents the results of demodulations available at the destination. The elements of $\mathbf{y}_{DemF}^{(S_i,R_j,D)}$ can take three possible values: $+1$ (representing a demodulated 0 bit), -1 (representing a demodulated 1 bit), and 0 (representing an unselected bit, akin to an erasure).

B. *Fractional Cooperation and Protocol*

Here we present a brief and simple protocol for fractional cooperation. (A similar protocol, minus the optimization, was implemented in hardware in [15].)

1) Each link is an orthogonal channel. The channel state information (CSI) of all the links are known to the destination, and the destination performs the optimization. For each S-R link, the relay estimates the SNR $\gamma_{(S_i,R_j)}$ and informs the destination; for the S-D and each R-D link, the destination estimates the SNR. The destination performs the optimization and transmits the required fractions $\epsilon_{(S_i,R_j)}$, for each source S_i, and the required R-D code rates, to every relay R_j. (This exchange is infrequent and involves little overhead if the environment is static or slow-moving).
2) Each source encodes its information using LDPC codes, and broadcasts the encoded codeword to the r relays, as well as the destination.
3) *(DemF)* For each source S_i, relay R_j observes m_i bits. The relay demodulates symbols from S_i, without decoding the underlying codewords.
4) *(Fractional Cooperation)* For each source S_i, R_j selects $m_i\epsilon_{(S_i,R_j)}$ of the demodulated bits for relaying (noting that $\epsilon_{(S_i,R_j)}$ is possibly zero). We assume the relays use a pseudorandom number generator to randomly select the symbols to be forwarded, with seed transmitted to the destination. Note that, aside from knowledge of the seed at the destination, there is no coordination of the selection among relays themselves.
5) The jth relay re-encodes the $m_i\epsilon_{(S_i,R_j)}$ bits using error correcting codes, at a rate specified by the destination, and transmits the resulting codeword to the destination. The destination then decodes each source's information

bits using the received signal from the r relays, and finally the source itself.

III. Linear Programming Model

In this section we present a linear programming model that minimizes the number of transmission bits of a multi-source multi-relay system, described in the previous section, subject to the constraint of successful transmission. We consider the multi-relay, multi-source system, explained in section II, with r relays and s sources. As is typical for analysis of LDPC decoding, we assume that the sources transmit the all-zero codeword (i.e., the all-(+1) channel codeword).

For codes with asymptotically long block length, the convergence of LDPC decoding is solely dependent on the distribution of the channel log-likelihood ratio (LLR) [13], given for a bit x by $\ell_x = \log f_{\mathbf{Y}}(\mathbf{y}|x=0)/f_{\mathbf{Y}}(\mathbf{y}|x=1)$, where $\mathbf{y}$ represents all observations of x available to the receiver. Letting $\gamma_{(S_i,D)}$ represent the SNR on the ith S-D link, the vector of channel LLRs for all symbols on the S-D links is calculated as

$$\boldsymbol{\ell}^{(S_i,D)} = 2\mathbf{y}^{(S_i,D)}/\sigma^2_{(S_i,D)} = 4\gamma_{(S_i,D)}\mathbf{y}^{(S_i,D)}, \tag{7}$$

and for the S-R links as

$$\boldsymbol{\ell}^{(S_i,R_j,D)}_{DemF} = \mathbf{y}^{(S_i,R_j,D)}_{DemF} \log\left[\frac{1-p^{(S_i,R_j)}_{Dem}}{p^{(S_i,R_j)}_{Dem}}\right], \tag{8}$$

where $p^{(S_i,R_j)}_{Dem}$ is the probability of demodulation error between the ith source and jth relay, given by

$$p^{(S_i,R_j)}_{Dem} = \frac{1}{2}\text{erfc}\left(\sqrt{\gamma_{(S_i,R_j)}}\right). \tag{9}$$

LLRs of independent observations of a symbol, such as those passed along different relay links, are additive. Consequently, the message LLR input to the iterative LDPC decoder of the ith source can be calculated as

$$\boldsymbol{\ell}^{(S_i)}_{DemF} = \boldsymbol{\ell}^{(S_i,D)} + \sum_{j=1}^{r}\boldsymbol{\ell}^{(S_i,R_j,D)}_{DemF}. \tag{10}$$

As is assumed in the EXIT chart literature, in (10) we assume that the distribution of $\boldsymbol{\ell}^{(S_i)}_{DemF}$ can be approximated by the Gaussian distribution. (This assumption is reasonable if the number of component messages r is sufficiently large, but high accuracy is *not* essential here: analysis of LDPC decoding under this assumption is known to be robust even if the true distribution is quite far from Gaussian; cf. [8].) Furthermore, a property of Gaussian-distributed LLRs is that the distribution is *symmetric* [14], with variance equal to twice the mean. Thus, we may use the mean of the LLR messages to represent their distribution, and hence to determine whether LDPC coding converges or not: this is the key observation which allows us to use linear programming.

Assuming a symmetric Gaussian-distributed LLR, a minimum channel LLR mean is required for successful decoding, written $m_{\ell_{min}}$. This value can be calculated using EXIT chart analysis, density evolution, or simulation. Let m_ℓ represent the mean of the channel LLR messages; then the constraint

$$m_\ell \geq m_{\ell_{min}} \tag{11}$$

is (approximately) sufficient to ensure successful decoding. This mean may be different for each source, so we will write $m^{(i)}_{\ell_{min}}$ to represent the minimum LLR for the ith source.

The following result gives the calculation of the input LLR mean to the decoder for DemF.

Proposition 1: For the system described in section II, assuming that the relays use DemF, the channel mean that is input to the iterative decoder for the ith source, S_i, is given by

$$\begin{aligned} m^{(DemF)}_{\ell_i} =& 2\gamma_{(S_i,D)} + \\ & \sum_{j=0}^{r}\epsilon_{(S_i,R_j)}(1-2p^{(S_i,R_j)}_{Dem})\log\left[\frac{1-p^{(S_i,R_j)}_{Dem}}{p^{(S_i,R_j)}_{Dem}}\right], \end{aligned} \tag{12}$$

where $\gamma_{(S_i,D)}$ is the channel SNR between the ith source and the destination, $p^{(S_i,R_j)}_{Dem}$ is the probability of hard decision error at the relay given by equation (9) and $\epsilon_{(S_i,R_j)}$ the fraction selected by each relay.

Proof: Taking the expected value of (10), since we have assumed that the transmission bits over R-D links are decoded successfully, we have $m^{(DemF)}_{\ell_i} = m^{(S_i,D)}_{\ell} + \sum_{j=0}^{r} m^{(S_i,R_j)}_{\ell}$. For the single S-D link the channel LLR mean is calculated as $m^{(S_i,D)}_{\ell} = 2\gamma_{(S_i,D)}$, where $\gamma_{(S_i,D)}$ is the channel SNR of the S-D link for the ith source, which gives the first term. Now, $m^{(S_i,R_j)}_{\ell}$ depends on the crossover probability $p^{(S_i,R_j)}_{Dem}$ on the (S_i,R_j) link, as well as the probability of selection $\epsilon_{(S_i,R_j)}$. If all bits are relayed,

$$m^{(S_i,R_j)}_{\ell} = (1-2p^{(S_i,R_j)}_{Dem})\log\left[\frac{1-p^{(S_i,R_j)}_{Dem}}{p^{(S_i,R_j)}_{Dem}}\right]. \tag{13}$$

In fractional cooperation, unselected positions have zero LLR (like an erasure). Thus,

$$m^{(S_i,R_j)}_{\ell} = \epsilon_{(S_i,R_j)}(1-2p^{(S_i,R_j)}_{Dem})\log\left[\frac{1-p^{(S_i,R_j)}_{Dem}}{p^{(S_i,R_j)}_{Dem}}\right], \tag{14}$$

and the proposition follows. ■

We are now ready to set up the linear program. In our model the objective is to minimize the number of transmitted bits (equivalent to minimizing energy) with the constraint of successful decoding at the destination. The objective variables are $\epsilon_{(S_i,R_j)}$, the forwarding fractions of relays. Define the objective vector of length sr, $\boldsymbol{\epsilon}$, as

$$\boldsymbol{\epsilon} = \left[\epsilon_{(1,1)} \ \cdots \ \epsilon_{(1,r)} \ \epsilon_{(2,1)} \ \cdots \ \epsilon_{(2,r)} \ \cdots \ \epsilon_{(s,1)} \ \cdots \ \epsilon_{(s,r)}\right]^T, \tag{15}$$

representing the fractions that are selected for transmission by each relay for each source.

The objective function gives the total energy consumption of the system. If DemF is used by the relays, and the ith source has a codeword of length m_i to transmit to the destination, the total energy consists of the energy required for the sources to transmit (proportional to m_i for the ith source), plus the energy committed by the relays to forward a fraction of each source (for the ith source and jth relay, proportional to $\epsilon_{(S_i,R_j)}m_i/r_{i,j}$, where $r_{i,j}$ is the rate of the jth relay's code).

Thus, the objective function is given by

$$f(\boldsymbol{\epsilon}) = \sum_{i=0}^{s} m_i + \sum_{i=0}^{s}\sum_{j=0}^{r} \frac{\epsilon_{(S_i,R_j)} m_i}{r_{i,j}}. \tag{16}$$

Since in Section II we assumed that powerful capacity approaching codes are used over the R-D links to ensure successful decoding at the destination, we can replace $r_{i,j}$ with the capacity of the corresponding channel. We can also omit terms not in $\boldsymbol{\epsilon}$ since they have no effect on the optimization. Therefore, the objective function becomes

$$f(\boldsymbol{\epsilon}) = \sum_{i=0}^{s}\sum_{j=0}^{r} \frac{\epsilon_{(S_i,R_j)} m_i}{C(\gamma_{(R_j,D)})}, \tag{17}$$

where $C(\gamma_{(R_j,D)})$ is the channel capacity between jth relay and the destination. Note that $f(\boldsymbol{\epsilon})$ is linear in $\boldsymbol{\epsilon}$.

To derive the constraints for DemF, we define a variable $g^{(S_i,R_j)}$ as

$$g_{(S_i,R_j)} = (1 - 2p_{Dem}^{(S_i,R_j)}) \log\left[\frac{1 - p_{Dem}^{(S_i,R_j)}}{p_{Dem}^{(S_i,R_j)}}\right], \tag{18}$$

where the term on the right side is derived in equation (13), and represents the R-D link channel LLR mean before fractional selection at the relays. An $s \times (sr)$ matrix, G_{SR}, is defined such that the s rows of the matrix represent the sources, and the sr columns represent the S-R channels. The columns are listed in the order of $(S_1,R_1)\ \cdots\ (S_1,R_r)\ (S_2,R_1)\ \cdots\ (S_2,R_r)\ \cdots\ (S_s,R_r)$, which represents the relays 1 through r forwarding for the first source, and then for the second source, and so on. For the ith, row the only nonzero elements are columns (S_i,R_1) to (S_i,R_r), where the values are $g_{(S_i,R_1)}$ to $g_{(S_i,R_r)}$ respectively. Therefore, using the row vector $\mathbf{g}_{i,SR} = [g_{(S_i,R_1)}, g_{(S_i,R_2)}, \ldots, g_{(S_i,R_r)}]$, the matrix G_{SR} is then given by

$$G_{SR} = \begin{bmatrix} \mathbf{g}_{1,SR} & \mathbf{0}_r & \cdots & \mathbf{0}_r \\ \mathbf{0}_r & \mathbf{g}_{2,SR} & \cdots & \mathbf{0}_r \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_r & \mathbf{0}_r & \ldots & \mathbf{g}_{s,SR} \end{bmatrix}, \tag{19}$$

where $\mathbf{0}_r$ is a row vector of r zeros. Let $\mathbf{m}_{\ell_{min}} = [m_{\ell_{min}}^{(1)}\ m_{\ell_{min}}^{(2)}\ \cdots\ m_{\ell_{min}}^{(s)}]^T$ represent the vector of minimum LLR means to ensure successful decoding. Also let $\boldsymbol{\gamma}_{SD} = [\gamma_{(S_1,D)}\ \gamma_{(S_2,D)}\ \cdots\ \gamma_{(S_s,D)}]^T$ represent the vector of S-D channel SNRs. Then, from Proposition 1, the successful decoding constraint may be stated in terms of $\boldsymbol{\epsilon}$ as

$$G_{SR}\boldsymbol{\epsilon} \geq \mathbf{m}_{\ell_{min}} - 2\boldsymbol{\gamma}_{SD}, \tag{20}$$

Further constraints are required on $\boldsymbol{\epsilon}$ to obtain a meaningful result, namely that

$$0 \leq \epsilon_{(S_i,R_j)} \leq \epsilon'_{(S_i,R_j)}, \tag{21}$$

where $\epsilon'_{(S_i,R_j)} \leq 1$. Additional constraints may also be added, depending on the application. Given the objective function in (17) and constraints in (20)-(21), the linear program may be stated completely as follows:

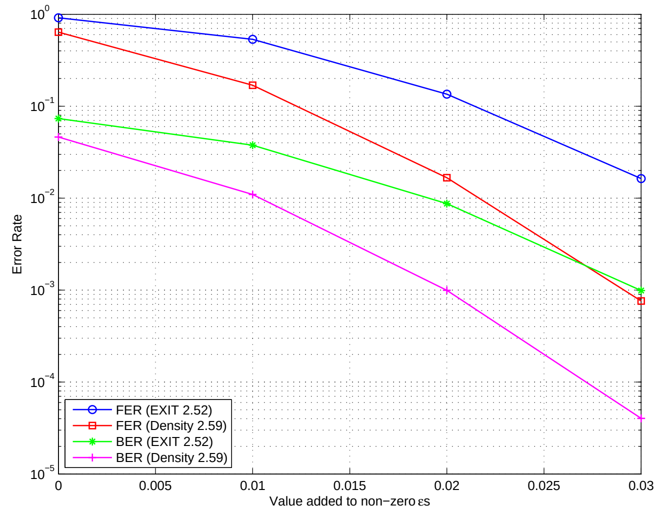

Fig. 2. Average frame error rate (FER) and bit error rate (BER) for 5 source, 50 relay system versus the value added to the non-zero fractions. FER and BER are averaged over all 5 sources. All sources use regular (3,6) LDPC code with codeword length of 10,000. The normalized SNR of all links are derived from Rayleigh distributed random variable. Both EXIT chart threshold ($m_{\ell_{min}} = 2.52$) and density evolution threshold ($m_{\ell_{min}} = 2.59$) are considered.

- Minimize

$$f(\boldsymbol{\epsilon}) = \sum_{i=0}^{s}\sum_{j=0}^{r} \frac{\epsilon_{(S_i,R_j)} m_i}{C(\gamma_{(R_j,D)})} \tag{22}$$

subject to

$$G_{SR}\boldsymbol{\epsilon} \geq \mathbf{m}_{\ell_{min}} - 2\boldsymbol{\gamma}_{SD}; \quad \boldsymbol{\epsilon} \geq \mathbf{0}; \quad \boldsymbol{\epsilon} \leq \boldsymbol{\epsilon}', \tag{23}$$

where $\mathbf{0}$ is an all-zero vector the same length as $\boldsymbol{\epsilon}$, and $\boldsymbol{\epsilon}'$ is the vector of $\epsilon'_{(S_i,R_j)}$ corresponding to $\boldsymbol{\epsilon}$.

IV. Results

For all of our simulations we use a (3,6) regular LDPC code at every source as well as a similar codeword length. To illustrate the sensitivity of our method to an accurate value of $m_{\ell_{min}}$, we present results deriving $m_{\ell_{min}}$ from both EXIT charts and density evolution: we have $m_{\ell_{min}} = 2.52$ and $m_{\ell_{min}} = 2.59$ for EXIT charts and density evolution, respectively. The codeword length is 10,000 bits, and reported bit error rate (BERs) / frame error rates (FERs) are averaged over all sources.

For our first set of simulations, we consider a cooperative scheme with 5 sources and 50 relays, and we assume the relays' transmission bits are perfectly decoded at the destination. We use a channel where every link has an independent Rayleigh-distributed signal strength; thus, the channel SNRs γ on all the links are independent, identically χ^2-distributed random variables with two degrees of freedom. For these results, we use $f_\Gamma(\gamma) = 1/0.1265 \exp(-\gamma/0.1265)$, giving an average SNR of -8.98dB. Also, we set $\epsilon'_{(S_i,R_j)} = 0.25$ instead of 1 (i.e., each relay may be unwilling to forward all the bits it receives), and add an extra constraint as $\sum_{i=1}^{5} \epsilon_{(S_i,R_j)} \leq 1$ (i.e., the relay's maximum relaying commitment does not exceed the equivalent of a single source). Figure 2 shows the

TABLE I

VALUES OF ϵ BASED ON EXIT CHART AND DENSITY EVOLUTION THRESHOLDS.

Relay	S_1(EXIT, Dens)	S_2(EXIT, Dens)	S_3(EXIT, Dens)	S_4(EXIT, Dens)	S_5 (EXIT, Dens)
R_4	(0 | 0)	(0 | 0.25)	(0 | 0)	(0.25 | 0.25)	(0.0980 | 0.25)
R_{16}	(0.0984 | 0.0963)	(0.25 | 0.25)	(0.0430 | 0.1537)	(0.25 | 0.25)	(0.25 | 0.25)
R_{20}	(0 | 0.25)	(0 | 0)	(0.25 | 0.25)	(0 | 0)	(0 | 0.0057)
R_{26}	(0.25 | 0.25)	(0.25 | 0.25)	(0.25 | 0.1568)	(0 | 0.0932)	(0.25 | 0.25)
R_{32}	(0| 0.0229)	(0.25 | 0.25)	(0 | 0)	(0.25 | 0.25)	(0 | 0)
R_{38}	(0.25 | 0.25)	(0 | 0.2447)	(0.25 | 0.25)	(0.25 | 0.25)	(0.25 | 0.0053)

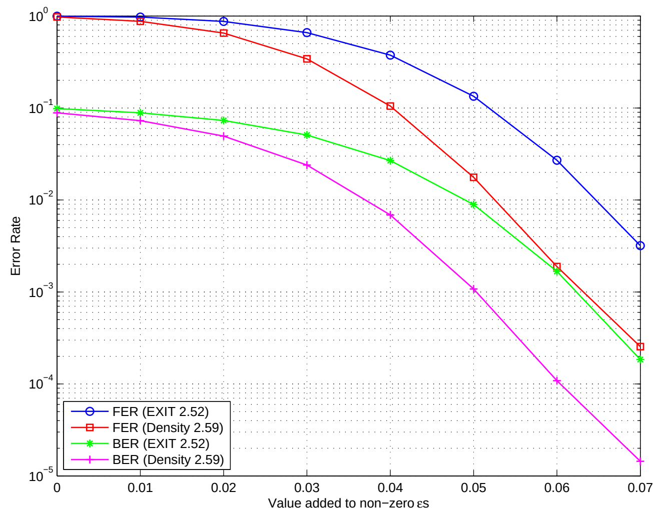

Fig. 3. Average frame error rate (FER) and bit error rate (BER) for 2 source, 7 relay system versus the value added to the non-zero fractions. R-D links' transmission symbols are encoded using PSRA codes with rates selected based on the channel SNR. FER and BER are averaged over the 2 sources. All sources use regular (3,6) LDPC code with codeword length of 10,000. The normalized SNR of all links are derived from Rayleigh distributed random variable. Both EXIT chart threshold ($m_{\ell_{min}} = 2.52$) and density evolution threshold ($m_{\ell_{min}} = 2.59$) are considered.

average BER and FER versus the value that is added to non-zero $\epsilon_{(S_i,R_j)}$ that were obtained using the linear programming model.

Table I illustrates the fractions calculated by our linear programming model for the 5 source, 50 relay system considered in Figure 2. We observe that not all the relays are forwarding the maximum fraction of 0.25; for instance, the density evolution results for sources 1, 3, and 5 show fractions being split among multiple relays. Thus, in general, the optimal strategy is *not* equivalent to relay selection.

For our second set of simulations, we consider a cooperative scheme with 2 sources and 7 relays. Here, we use PSRA codes over realistic R-D links, where the code rates are selected based on the density evolution threshold of the PSRA code for a given channel SNR. For these results, we use $f_\Gamma(\gamma) = \exp(-\gamma)$, giving an average SNR of 0dB. Also, we set $\epsilon'_{(S_i,R_j)} = 0.25$, and add an extra constraint as $\sum_{i=1}^{5} \epsilon_{(S_i,R_j)} \leq 0.4$ (i.e., the relay's maximum relaying commitment does not exceed the equivalent of a 0.4 source). Figure 3 shows the average frame error rate (FER) and bit error rate (BER) of the system versus the value that is added to non-zero $\epsilon_{(S_i,R_j)}$ that were obtained using the linear programming model. We see that the BER and FER waterfalls start right at the predicted threshold.

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